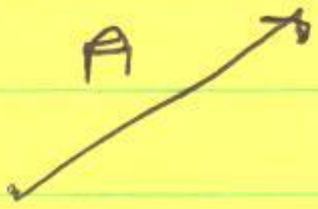
1

Vectors

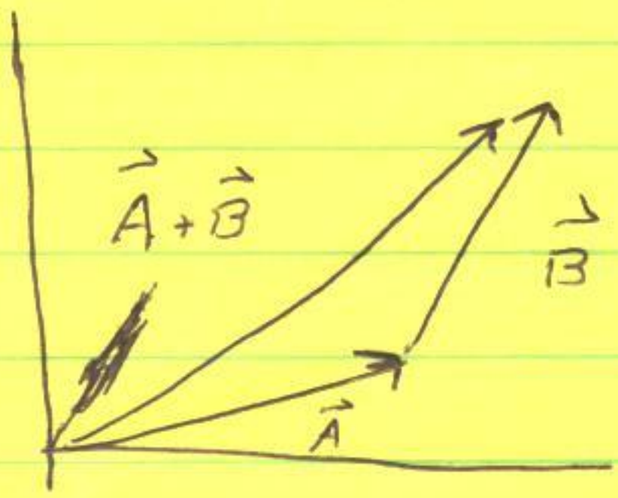
(1) Have Magnitude & Direction



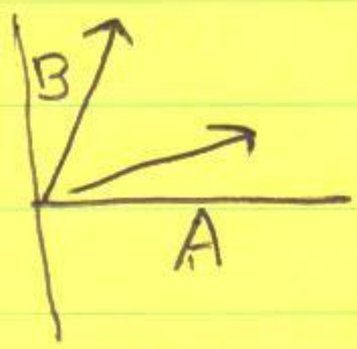
Vectors : velocity, acceleration,

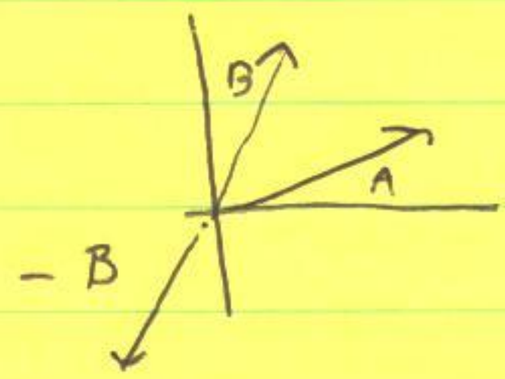
Scalars : mass, time, speed

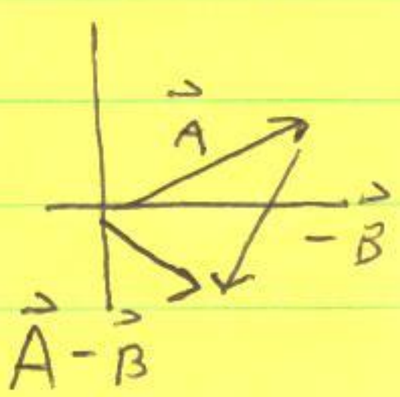
(2) Add Them tip to tail



(3) Subtract :

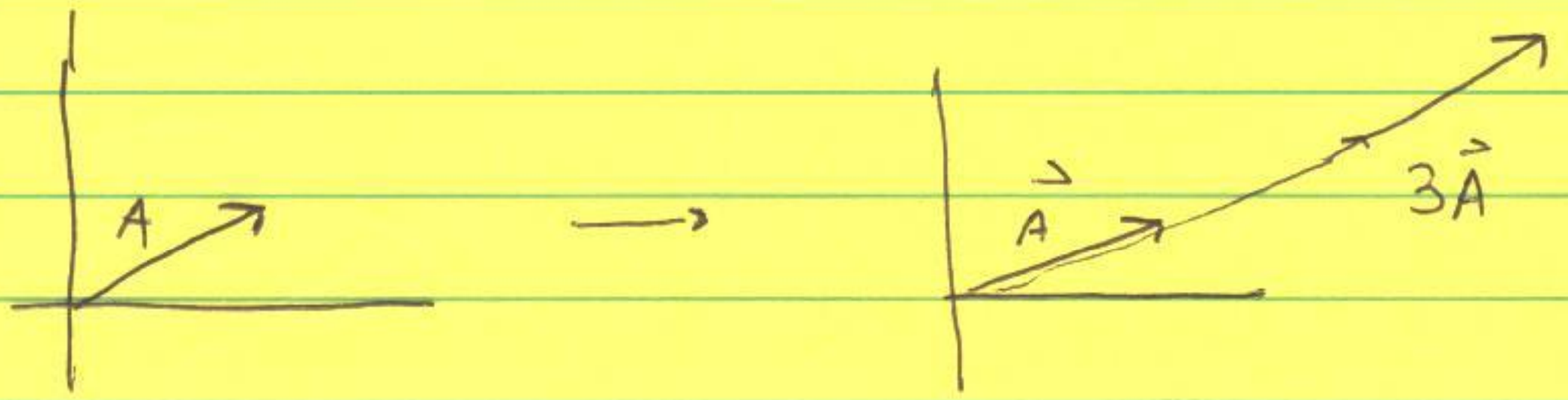




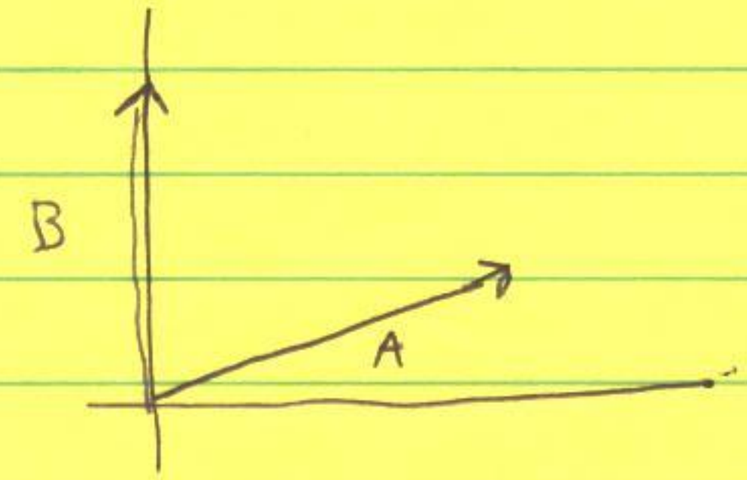


(2)

(4) Multiply by a number = scalar



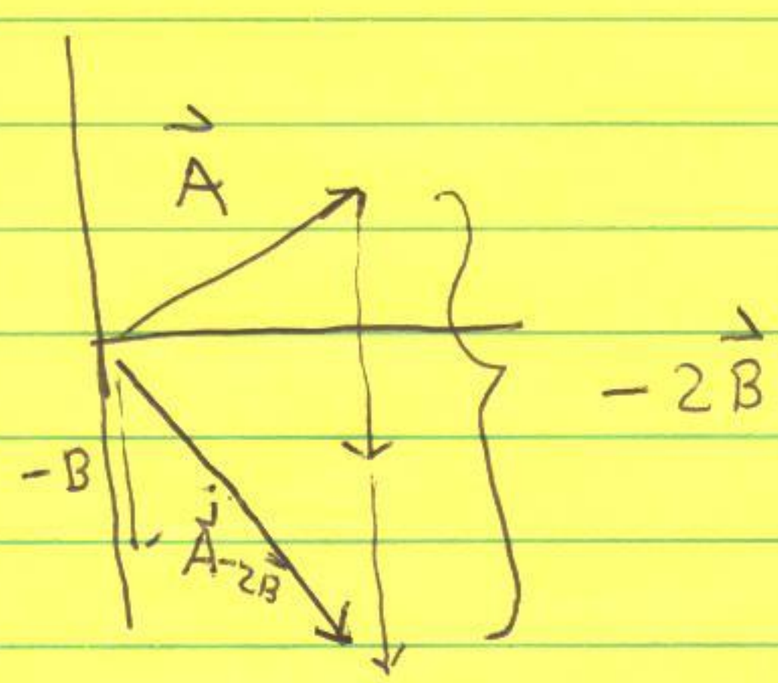
Problem: PSE #15



(1) Find $(\vec{A} - 2\vec{B})$ $\vec{B} - 2\vec{A}$

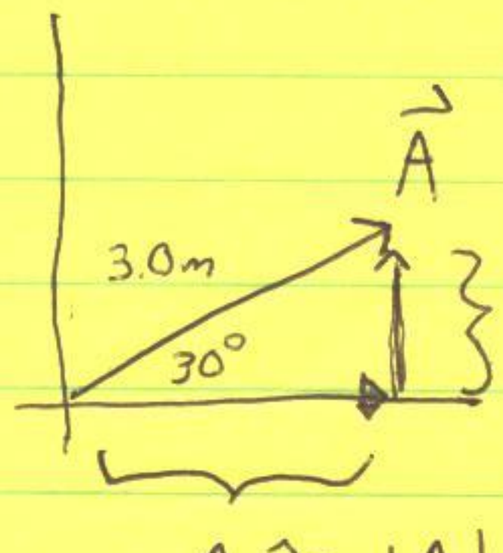
$\vec{A} + \vec{B}$

Ex



(3)

Coordinates



$A_y \hat{j} = |A| \sin\theta \, \hat{j}$

$A_x \hat{i} = |A| \cos\theta \, \hat{i}$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\vec{A} = 2.59 \hat{i} + 1.5 \hat{j}$$

Alternate Notation:

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 2.59 \\ 1.5 \end{pmatrix}$$

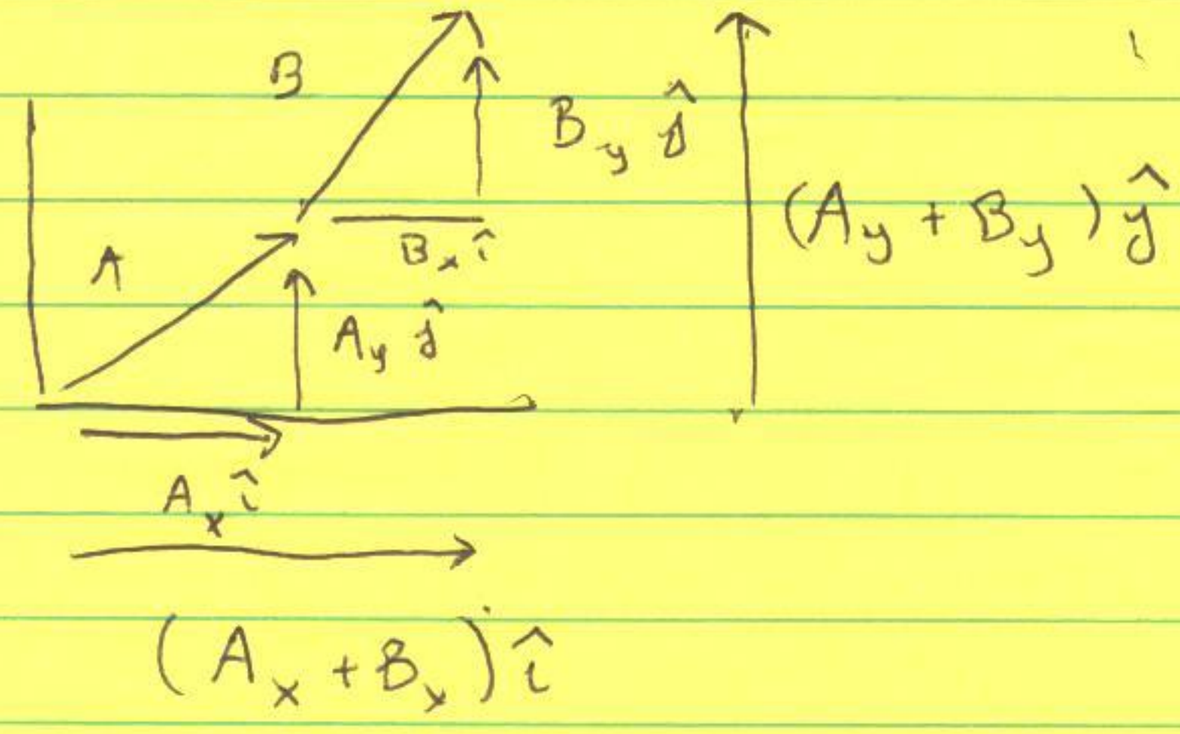
means same thing!

Ex Write $\vec{B}$ in our notations

$\vec{B} = 3.0 \text{ m} \, \hat{j}$

$$\vec{B} = \begin{pmatrix} 0 \\ 3.0 \end{pmatrix}$$

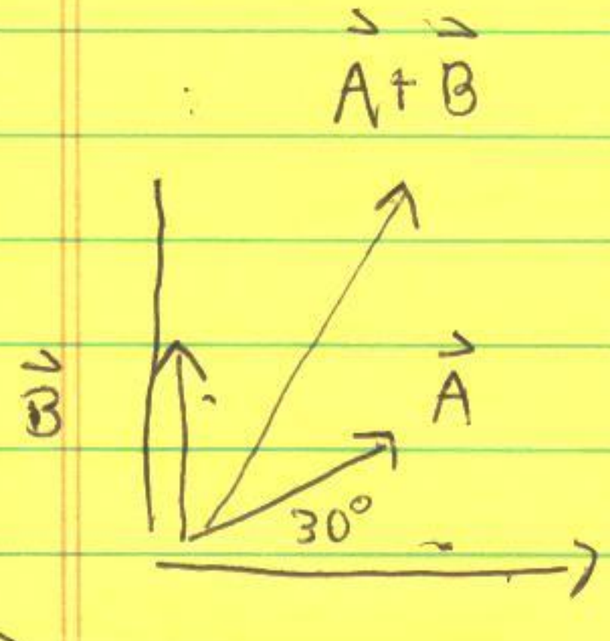
Adding Vectors



$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} + \vec{B} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} A_x + B_x \\ A_y + B_y \end{pmatrix}$$

Ex :



Add These two vectors algebraically.

$$\vec{A} + \vec{B} = (2.59\hat{i} + 1.5\hat{j}) + (3.0\hat{j})$$

$$= (2.59\hat{i}) + 4.5\hat{j}$$

(5)

Find the length of $|\vec{A} + \vec{B}|$ and angle relative to axes

Solution

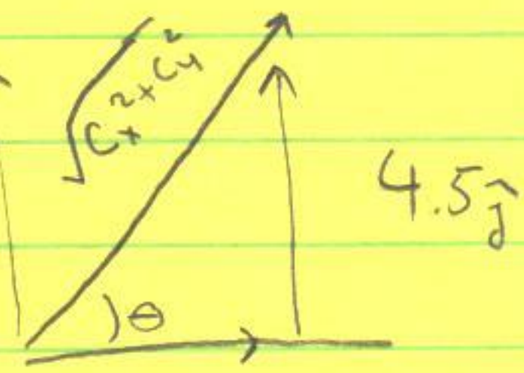
Length

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = (2.59\hat{i}) + 4.5\hat{j}$$

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{(2.6)^2 + (4.5)^2} = 5.2$$

Angle



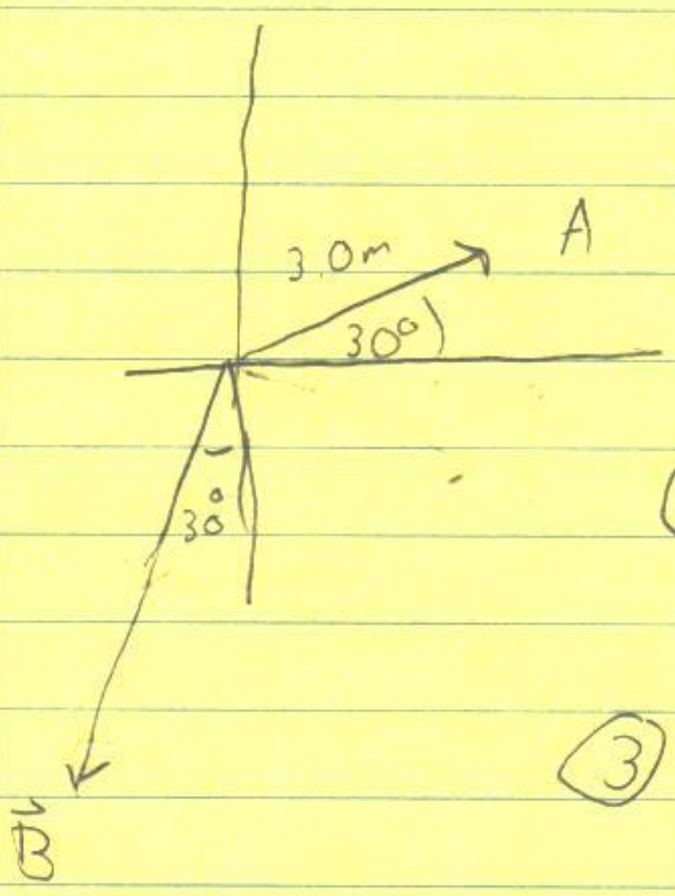
$$\sin\theta = \frac{4.5}{5.2} = 1.04 \text{ Rad}$$

$$\approx 60^\circ$$

$$\tan\theta = \frac{4.5}{2.59} \Rightarrow \theta = 60^\circ$$

Example: $A = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

$B = -\hat{i} + -\sqrt{3}\hat{j}$



(1) Draw $\vec{A}$ and $\vec{B}$

(2) Find the length of $|\vec{A}| + |\vec{B}|$

(3) Find $2\vec{A} - \vec{B}$, grapically & numerically

(4) Find $\frac{\vec{B}}{2} - \vec{A}$ find the angle between the "horizon" and this vector

(5) Find the length $\frac{\vec{B}}{2} - \vec{A}$

(6a)

Solution:

$$|A| = \sqrt{|A_x|^2 + |A_y|^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

(2)

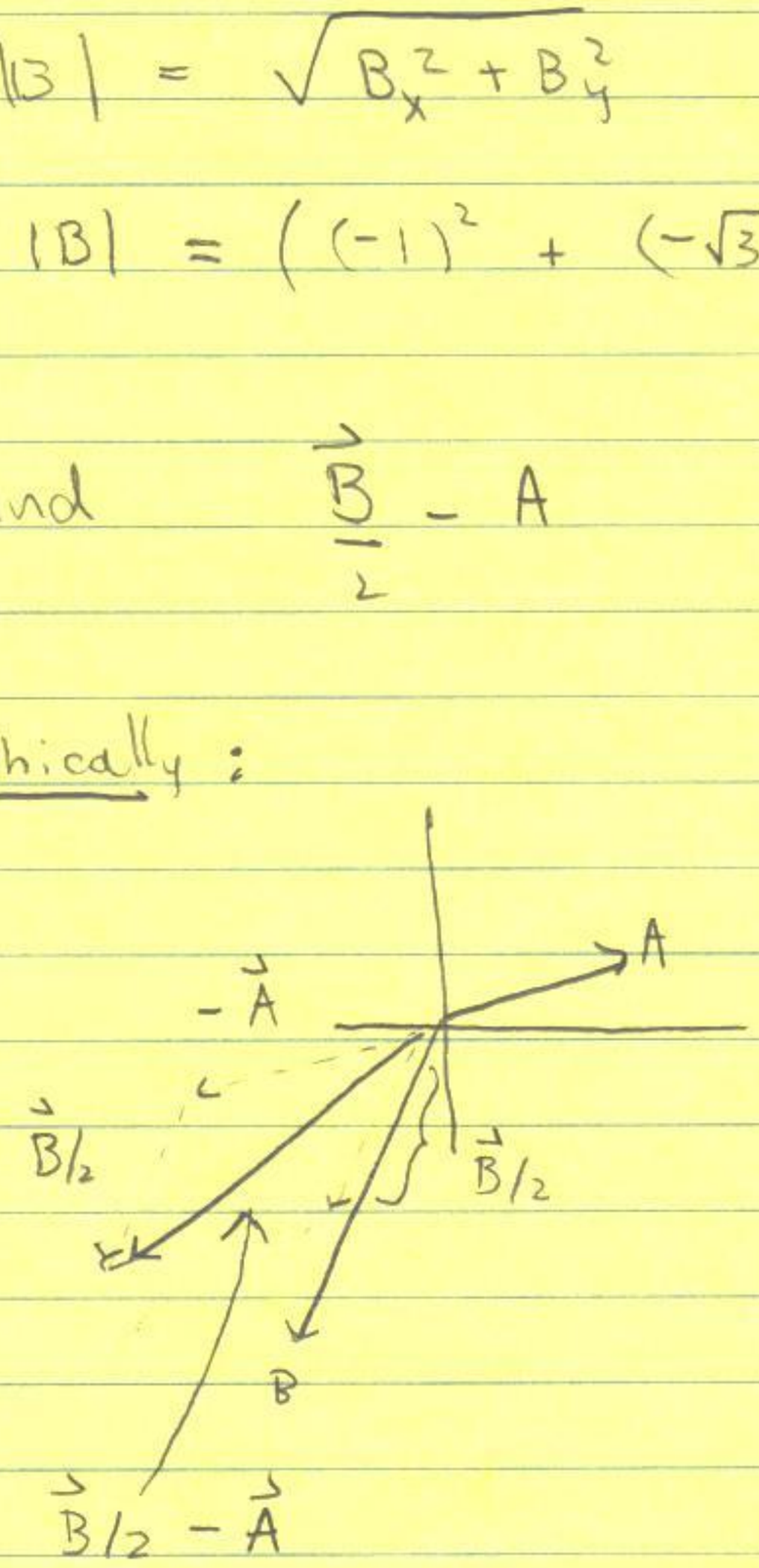
$$= \left(\frac{3}{4} + \frac{1}{4}\right)^{1/2} = 1 \quad \text{L}$$

$$|B| = \sqrt{B_x^2 + B_y^2}$$

$$|B| = \left((-1)^2 + (-\sqrt{3})^2\right)^{1/2} = 2 \quad \text{⌣}$$

(4) Find $\frac{\vec{B}}{2} - A$

Graphically:



Symbollically:

$$\vec{C} = \frac{1}{2}\vec{B} - \vec{A}$$

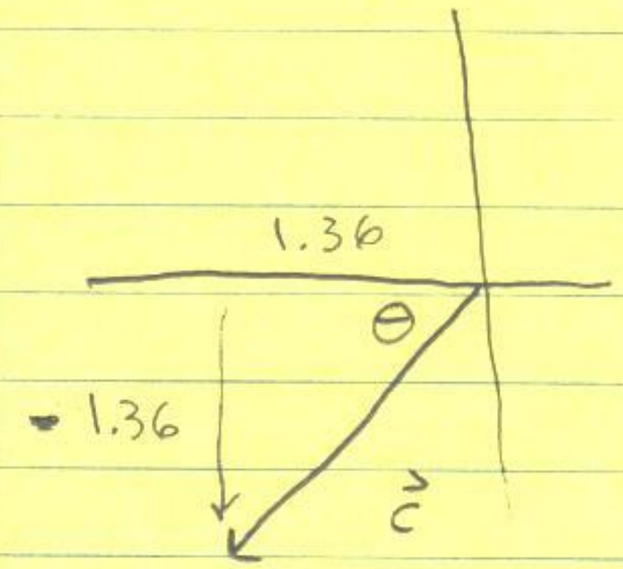
$$\vec{C} = \frac{1}{2}\begin{pmatrix} -1 \\ -\sqrt{3} \end{pmatrix} - \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$$

$$\vec{C} = \begin{pmatrix} -\frac{1}{2} - \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} - \frac{1}{2} \end{pmatrix}$$

$$\vec{C} = \begin{pmatrix} -(1+\sqrt{3})/2 \\ -(1+\sqrt{3})/2 \end{pmatrix}$$

$$\vec{C} \simeq \begin{pmatrix} -1.36 \\ -1.36 \end{pmatrix}$$

6b



$$|c| = \sqrt{(1.36)^2 + (1.36)^2}$$

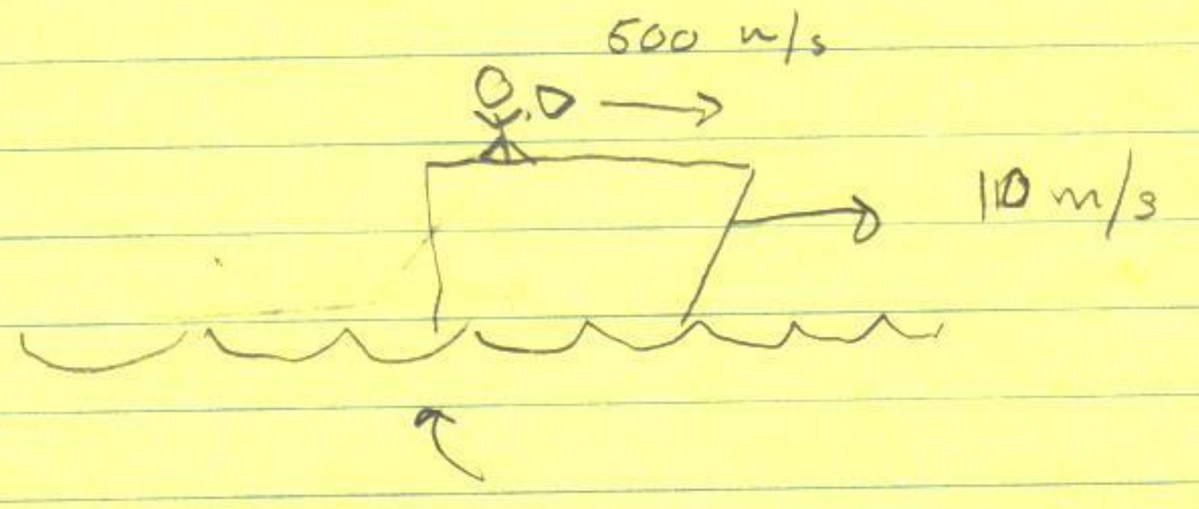
$$|c| = 1.92$$

$$\tan\theta = \frac{-1.36}{-1.36} = 1 \qquad \theta = 45^\circ$$

(7)

Addition of Velocities :

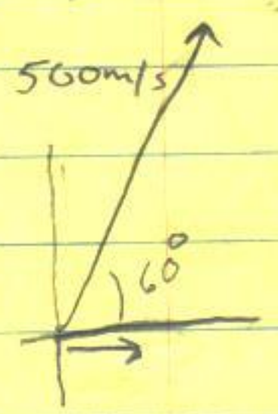
$$\vec{V}_{\text{bullet}|\text{earth}} = \vec{V}_{\text{bullet}|\text{boat}} + \vec{V}_{\text{boat}|\text{earth}}$$



$V_{\text{Bullet}|\text{earth}} = 510 \text{ m/s}$

$\vec{V}_{\text{Bullet}|\text{earth}} = ?$ ⇒ Use vectors

Graphically



$\vec{V}_{\text{Boat}|\text{earth}}$

(A) $\vec{V}_{\text{Bullet}|\text{Boat}} = \underbrace{500 \text{ m/s} \cos 60^\circ}_{250 \text{ m/s}} \hat{i} + \underbrace{500 \text{m/s} \sin 60^\circ}_{433 \text{ m/s}}$

(B) $\vec{V}_{\text{Boat}|\text{earth}} = 10 \text{ m/s } \hat{i}$

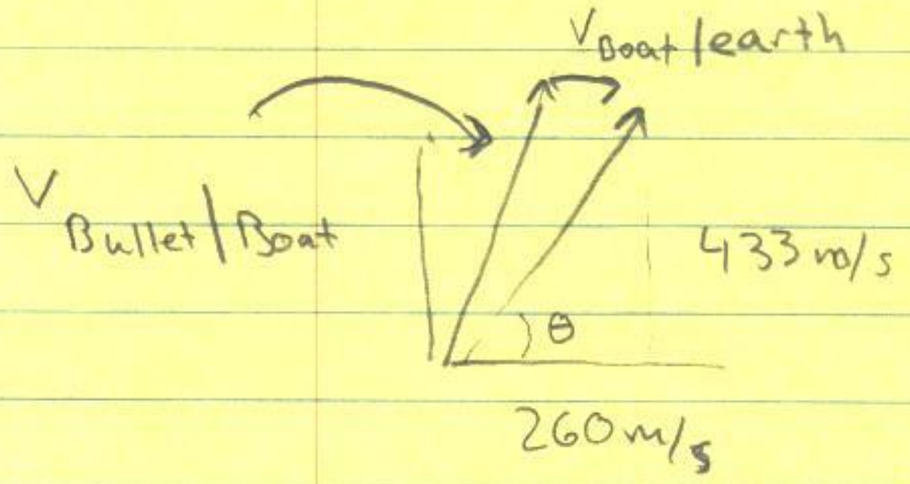
$\vec{V}_{\text{Bullet}|\text{earth}} = (A) + (B)$

$= 260 \text{ m/s } \hat{i} + 433 \text{ m/s}$

$|V_{\text{Bullet}|\text{earth}}| = \sqrt{(260)^2 + (433)^2} = 505 \text{ m/s}$

(8)

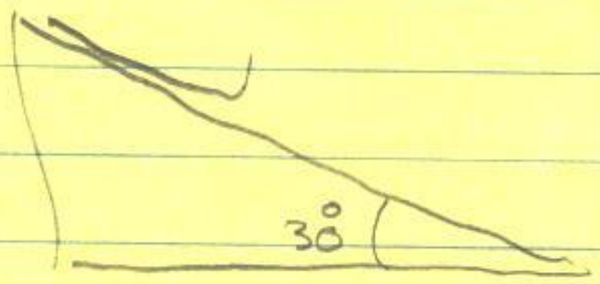
Angle



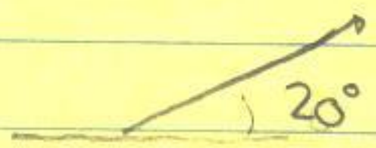
$$\tan\theta = \frac{433\ m/s}{260 m/s} \Rightarrow \theta = 59^\circ$$

(9)

Boy travelling on a sled down a slope



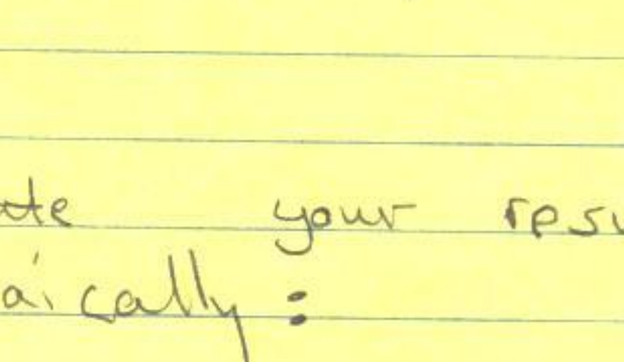
He mounts a BB gun to his sled at a 20° angle relative to the horizon



which fires BB's with speed 7m/s when he is travelling at 5m/s what angle and speed does the BB come out.

Illustrate your result graphically and algebraically:

Graphically:



(1) The speed of the BB is faster

(2) The angle is greater than 10°